An Introduction to Symmetric Functions and Their Combinatorics
by Eric S. Egge
Corrections
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Below is a list of all of the corrections to my book *An Introduction to Symmetric Functions and Their Combinatorics* that I have received so far. This document has a companion (currently in progress) of additional clarifications and comments. I may have misunderstood (or mistyped) some of these corrections, so please let me know if something here looks incorrect.

I would like to thank everyone who has sent me corrections. I would especially like to thank Darij Grinberg for his helpful feedback, and for sharing his extensive list of corrections and comments.

- Page 2: in the paragraph before Section 1.1, "the set of all symmetric functions with coefficients in $\mathbb{Q}$ is a finite-dimensional vector space over $\mathbb{Q}$" should be "for each positive integer k, the set of all symmetric functions with coefficients in $\mathbb{Q}$ in which each term has total degree k is a finite-dimensional vector space over $\mathbb{Q}$".

- Page 3: in the paragraph after the solution to Example, 1.1, "for all $n \geq 1$ we will write X_n to denote the set of variables $x_1, \ldots, x_n$" should be "for all $n \geq 0$ we will write X_n to denote the set of variables $x_1, \ldots, x_n$. In particular, X_0 is the empty set."

- Page 14: in the definition of total degree in the middle of the page, $a_1 + a_2 \cdots$ should be $a_1 + a_2 + \cdots$.

- Page 17: between Definitions 1.19 and 1.20, "we constructed the monomial symmetric functions" should be "we constructed the monomial symmetric polynomials".

- Page 17: in the last two lines, "sum of the images of $x_1^2 x_2$ under all permutations of $x_1, \ldots, x_n$" should be "sum of the distinct images of $x_1^2 x_2$ under all permutations of $x_1, \ldots, x_n$".

- Page 24: in the paragraph after the solution to Example 2.1, "is the sum of all products" should be "is (up to sign) the sum of all products".

- Page 25: the last sentence of Definition 2.2 should be two sentences: "By convention $e_0(X_n) = 1$ for all $n \geq 0$, $e_0 = 1$, and if $n < k$ then $e_k(X_n) = 0$. In particular, $e_k(X_0) = 0$ for all $k \geq 1$."

- Page 30: in the fifth and sixth lines after Figure 2.4, "and one in case A4" should be "and one in case A3".

- Page 31: at the beginning of the proof of Proposition 2.10, "By (2.2)" should be "By the next displayed equation after (2.2)".

- Page 37: in the last three lines, "By Corollary 2.11 the elementary symmetric functions are linearly independent, so all of the coefficients in $f(y_1, \ldots, y_n)$ are equal to 0" should be "By Corollary 2.11 the elementary symmetric functions are linearly independent. Since different monomials in $f(y_1, \ldots, y_n)$ correspond to different elementary symmetric functions, all of the coefficients in $f(y_1, \ldots, y_n)$ are equal to 0."

- Page 39: at the beginning of Definition 2.19, "complete homogeneous polynomial" should be "complete homogeneous symmetric polynomial".

- Page 39: after the second displayed equation, the sentence beginning "By convention" should be "By convention, $h_0(X_n) = 1$ for all $n \geq 0$, $h_0 = 1$, and $h_k(X_0) = 0$ for all $k \geq 1$."

- Page 41: twice in the first two lines after the solution to Example 2.22, "Example 2.20" should be "Example 2.22".

- Page 49: in Problem 2.14, both instances of $n \geq 0$ should be $n \geq 1$.

- Page 50: in Problem 2.24 the exponent on x_j should be $n - 1 + k$ rather than $n - 1 - k$.

- Page 51: in Problem 2.28(a), $n \times n$ should be $k \times k$.

- Page 51: in Problem 2.28(a), "exactly one entry of column j is λ_j and all other entries in column j are 0" should be "column j has one entry equal to λ_j and all other entries in this column are 0".

- Page 51: in Problem 2.28(b), "for all j" should be "for each j".

- Page 55: in the fifth line, "when $k = 7$ and $j = 3$" should be "when $k = 7$ and $j = 4$".

- Page 57: in the proof of Proposition 3.3, "after the rightmost r" should be "after the rightmost number less than or equal to r".

- Page 58: the displayed equation between Propositions 3.5 and 3.6 should be

$$\sum_{j=1}^{m} (-1)^{j+1} \frac{n}{j} \binom{m+n-j-1}{j-1, n-j, m-j} = \binom{m+n-1}{m}.$$

- Page 59: in the first displayed equation, if any of the lower arguments of a multinomial coefficient is negative then we take that multinomial coefficient to be 0.

- Page 62: in the ninth line, "each permutation of $[n]$ with exactly k cycles" should be "each permutation of $[n]$ of the second type with exactly k cycles".

- Page 62: in the tenth line, "wth" should be "with".

- Page 63: on the right side of (3.20), $(-1)^{j-1}$ should be $(-1)^{j-1}j$.

- Page 64: on the left side of (3.21), $\begin{bmatrix} n \\ j \end{bmatrix}$ should be $\begin{bmatrix} n \\ k \end{bmatrix}$.

- Page 69: two lines above Proposition 3.27, $1 \cdot q \cdot q^2 \cdots q^k$ should be $1 \cdot q \cdot q^2 \cdots q^{k-1}$.

- Page 70: on the right side of (3.28), $\binom{n}{k}_q$ should be $\binom{n}{j}_q$.

- Page 71: in Problem 3.3, $\sum_{j=1}^{n}$ should be $\sum_{j=1}^{k}$.

- Page 75: at the start of Section 4.1, "In Chapter 2.16 and 2.17 we described" should be

In Chapter 2 we defined the elementary symmetric functions in terms of subsets of $[n]$, and the complete homogeneous symmetric functions in terms of multisubsets of $[n]$. We then gave combinatorial interpretations of the coefficients we obtain when we write the elementary symmetric functions as linear combinations of the monomial symmetric functions. And in Problems 2.16 and 2.17 we described

- Page 81: in the sixth and seventh lines after Figure 4.5, "seminstandard" should be "semistandard".

- Page 83: in line 15, "Therefore, in T we have" should be "Therefore, in R we have".

- Page 87: in the second line, "our solution to Example 4.12 raises" should be "our solution to Example 4.9 raises".

- Page 91: in the second displayed equation in the proof of Proposition 4.24, $s_{\tau(n)}^{\mu_n}$ should be $x_{\tau(n)}^{\mu_n}$.

- Page 92: in the first line, $n-1, n-2, \ldots, 2, 1$ should be $n-1, n-2, \ldots, 2, 1, 0$.

- Page 97: after the proof of Proposition 4.31, "the left side of (4.12)" should be "the left side of (4.13)".

- Page 100: in Definition 4.37, "For each entry $k_j = \pi(l_j)$" should be "For each entry $k_j = \pi(l_j)$ with $l_j > 1$", "which are equal to $\pi(l_j + 1)$" should be "which are equal to $\pi(l_j - 1)$", and $t_j > r_j$ should be $t_j < r_j$.

- Page 102: in the second line, "will not start with $\pi(1)$" should be "will not end with $\pi(1)$".

- Page 103: in Proposition 4.40, $l(\pi) \geq n$ should be $l(\pi) \leq n$.

- Page 103: in Proposition 4.40(ii), "is the leftmost entry" should be "is the rightmost entry".

- Page 104: in the third and fourth lines from the bottom of the page, "results follow" should be "result follows".

- Page 107: in the third paragraph of the solution to Example 4.42, "middle tableau" should be "second tableau from the left".

- Page 107: near the end of the third paragraph of the solution to Example 4.42, "This means $k = 1$, $j = 2$, and $l = 3$" should be "This means $k = 1$, $j = 3$, and $l = 3$".

- Page 108: in Lemma 4.43(ii), "not a semistandard π-tableau" should be "not a Littlewood-Richardson π-tableau".

- Page 111: near the middle of the page, "over all Littlewood-Richardson π-tableau" should be "over all Littlewood-Richardson π-tableaux".

- Page 113: in Problem 4.20, $n \geq k \geq 0$ should be $n > k \geq 0$.

- Page 116: in Problem 4.36, "Let $\tilde{\lambda}$ be the partition with" should be "Let $\tilde{\lambda}$ be the partition with at most n parts and with".

- Page 116: in Problem 4.36, λ_{r-j+1} should be λ_{n-j+1}.

- Page 121: shortly before (5.1), "By our definition of the skew Schur functions" should be "By our definition of the skew Schur functions (and by Proposition 5.4)".

- Page 126: in the seventh line of the proof of Proposition 5.11, "semistandard tableaux" should be "semistandard tableau".

- Page 127: in the third line from the end of the proof of Proposition 5.12, "in the its" should be "in its".

- Page 130: after the last displayed equation in Definition 5.16, "over all set-valued tableaux" should be "over all set-valued semistandard tableaux".

- Page 136: in the last line of Definition 5.22, "of elegant tableau" should be "of elegant tableaux".

- Page 136: in Example 5.24, $k \geq n$ should be $k \geq n > 0$.

- Page 136: in the last line on the page, $n + j$ should be $n + j - 1$.

- Page 137: in the second line on the page, $n + j$ should be $n + j - 1$.

- Page 138: in the first sentence after Figure 5.13, "reverse plane partitions with entries in $[n]$" should be "reverse plane partitions of shape λ with entries in $[n]$" and "reverse plane partitions with entries in $\mathbb{P}$" should be "reverse plane partitions of shape λ with entries in $\mathbb{P}$".

- Page 141: in the first bullet point, in "we replace each j with $j + 1$ in the left column and we replace each $j + 1$ with j in the right column", the words "left" and "right" should be interchanged.

- Page 142: in the eleventh line after Figure 5.17, $\sum_k jv_k$ should be $\sum_k kv_k$.

- Page 143: in Example 5.32, $n \geq 0$ should be $n \geq 1$.

- Page 143: in the solution to Example 5.32, "with at least one j for $1 \leq j \leq k$" should be "with at least one j for each $1 \leq j \leq k$".

- Page 146: in the third line before the end of the proof of Proposition 5.36, "the endpoint we removed" should be "the vertex we removed".

- Page 146: in the last line of the proof of Proposition 5.36, "for the color of the endpoint" should be "for the color of the remaining vertex".

- Page 146: Definition 5.37 should be for all colorings of G, rather than just for proper colorings.

- Page 148: in the first line of the proof of Proposition 5.41, $x_{\lambda_{l(\lambda)}}^{\lambda_{l(\lambda)}}$ should be $x_{l(\lambda)}^{\lambda_{l(\lambda)}}$.

- Page 148: in the first two lines of the proof of Proposition 5.41, "(and therefore the coefficient of m_λ in X_G)" should be "(and therefore the coefficient of m_λ) in X_G".

- Page 148: in the third line from the end of the proof of Proposition 5.41, "by choosing a stable partition $V_1, V_2, \ldots$ of G" should be "by choosing a stable partition $V_1, V_2, \ldots$ of G with $|V_j| = \lambda_j$ for all j".

- Page 155: the left side of the displayed equation in Problem 5.22 should be $\sum_{n=0}^{\infty} X_{P_n} t^n$.

- Page 156: in the third line of the last paragraph, "of graphs" should be "of connected graphs".

- Page 162: in the sixth line after Figure 6.4, "moves $\lambda_j + \pi_j - j$" should be "moves $\lambda_j + \pi(j) - j$".

- Page 162: in the last two lines, "ending j units from the right" should be "whose ending point is the jth one from the right".

- Page 168: at the start of the proof of Lemma 6.7, "Suppose $\beta \in H_{\lambda,k}$" should be "Suppose $\beta \in II_{\lambda,k}$".

- Page 173: at the end of the sixth line after Figure 6.13, "product of the weights" should be "product of the e-weights".

- Page 176: in the third line of the proof of Lemma 6.12, "leftmost point" should be "rightmost point".

- Page 176: in Lemma 6.14, SST(λ') should be SST(λ).

- Page 176: in the second line of the proof of Lemma 6.14, "Ferrers diagram of λ'" should be "Ferrers diagram of λ". (The correct Ferrers diagram is for λ, rather than for its conjugate.)

- Page 176: in the third line of the proof of Lemma 6.14, "weights" should be "e-weights".

- Page 176: in the fourth line of the proof of Lemma 6.14, "column strict" should be "column-strict".

- Page 178: the sum in the third displayed line in the proof of Theorem 6.10 should be over all $T \in \text{SST}(\lambda)$ rather than over all $T \in \text{SST}(\lambda')$.

- Page 178: in the last displayed line in the proof of Theorem 6.10, $s_{\lambda'}$ should be s_λ.

- Page 194: in the second line before Proposition 7.4, "inner product" should be "bilinear map on $\Lambda_k \times \Lambda_k$".

- Page 194: in both places in Proposition 7.4, "is the function" should be "is the bilinear map".

- Page 195: in the second displayed line in the proof of Proposition 7.4, "(by Definition A.15(i)–(iii))" should be "(by Definition A.15(1)–(3))".

- Page 195: in the seventh line of the proof of Proposition 7.4, "unique function" should be "unique bilinear map".

- Page 195: in the second displayed line in the proof of (i) $\Leftrightarrow$ (iii) for Proposition 7.4, $B_{\mu\beta}$ should be $B_{\lambda\beta}$.

- Page 202: in the second line of the solution to Example 7.16, $h_0 = p_0$ should be $h_0 = p_\emptyset$, where $\emptyset$ is the partition with no parts.

- Page 206: in the seventh line after Figure 7.6, "for any $n \geq 0$" should be "for any $n \geq 1$".

- Page 226: in the line before Definition 8.12, $x^{Q(\pi)} = x^{\text{topwt}(\pi)}$ should be $x^{Q(\pi)} = \text{topwt}(\pi)$.

- Page 231: in the last sentence of Theorem 8.19, $x^{\text{topwt}(\pi)}y^{\text{bottomwt}(\pi)} = x^{Q(\pi)}y^{P(\pi)}$ should be "topwt$(\pi) = x^{Q(\pi)}$ and bottomwt$(\pi) = y^{P(\pi)}$".

- Page 242: in the proof of Theorem 8.29, "over a line from the lower left corner to the upper right corner" should be "over the line of slope 1 passing through the lower left corner".

- Page 242: in the proof of Theorem 8.29, "the P-labeling and the Q-labeling" should be "the P-filling and the Q-filling".

- Page 243: in Problem 8.10, "whose last entry is a_j" should be "whose last term is the term in the jth position of $a_1, \ldots, a_n$".

- Page 246: in the fourth line, "hook lengths" should be "hook-lengths", and in the ninth line "hook length" should be "hook-length".

- Page 248: in the fourth line of Section 9.1, $s_{11} = h_2$ should be $s_{11} = e_2$, and $s_2 = e_2$ should be $s_2 = h_2$.

- Page 252: in the eighth and ninth lines of the proof of Proposition 9.4, "we can group fillings of ν/μ of content λ" should be "we can group the semistandard tableaux of shape ν/μ and content λ".

- Page 254: in the first displayed line,

$$\sum_{\zeta} K_{\nu,\zeta} m_\nu$$

should be

$$\sum_{\zeta} K_{\nu,\zeta}, m_\zeta.$$

- Page 260: the outer sum in (9.5) should be

$$\sum_{\substack{\mu_{vh} \supseteq \mu \\ |\mu_{vh}/\mu| = n}}$$

- Page 264: in the tenth line of the proof of Lemma 9.16, "is to the left (resp., below)" should be "is to the left of (resp., below)".

- Page 265: in the first line, "Lemma 9.16 tells" should be "Lemmas 9.15 and 9.16 tell".

- Page 265: the outer sums in lines (9.6) and (9.7) should be

$$\sum_{\substack{\mu_{vh} \supseteq \mu \\ |\mu_{vh}/\mu| = n \\ \mu_{vh}/\mu \text{ is a border strip}}} .$$

- Page 265: in Theorem 9.17, "For all $n \geq 0$" should be "For all $n \geq 1$".

- Page 266: in fourth line of Definition 9.18, "for all j with $0 \leq j \leq l(\mu)$" should be "for all j with $0 \leq j \leq l(\mu) - 1$". Similarly, the upper bound on the displayed product should be $l(\mu) - 1$ rather than $l(\mu)$.

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- Page 267: in the fourth line of the proof of Theorem 9.20, "is a hook $n - j, 1^j$" should be "is a hook $(n - j, 1^j)$".

- Page 270: in Problem 9.10, "into (nonempty) border strips" should be "into (nonempty) connected border strips".

- Page 272: the right side of (10.3) is missing $s_{411}(X_3)$, so it should be

$$s_{42}(X_3) + s_{411}(X_3) + s_{33}(X_3) + 2s_{321}(X_3) + s_{222}(X_3).$$

- Page 273: in the first two lines after Table 10.1 the list of shapes should also include $(4, 1, 1)$.

- Page 274: in line 12, "row nondecreasing" should be "row-nondecreasing".

- Page 278: in the second line of the second paragraph of Section 10.2, "suppose word(T_1)" should be "suppose word(T_2)".

- Page 279: in line 12, "for some $i \geq j$" should be "for some $i > j$".

- Page 279: in the third line before Definition 10.8, $y = a_{i+1}$ should be $y = \begin{cases} a_{i+1} & \text{if } i < j - 1; \\ c & \text{if } i = j - 1 \end{cases}$.

- Page 281: in the fourth line after Figure 10.7, "a set of words are exactly" should be "a set of words closed under elementary Knuth transformations are exactly".

- Page 281: in the third line from the end of the page, "as a product on words" should be "as a product on tableaux".

- Page 282: in the line before Corollary 10.13, "tableaux" should be "tableau".

- Page 283: in the second line of the proof of Theorem 10.14, "using using" should be "using".

- Page 286: in the fifth line after Figure 10.10, "identical to the left of x and to the right of z" should be "identical to the left and to the right of the respective yzx or yxz blocks".

- Page 289: in the sixth line of the proof of Theorem 10.17, "to its right" should be "to its left".

- Page 292: in the seventh line, "tableaux" should be "tableau".

- Page 296: in the third line of the proof of Lemma 10.29, "with b removed" should be "with the leftmost appearance of b removed in each word".

- Page 298: in the second line of the proof of Theorem 10.33, "with $P(\sigma) = V$, and let $Q(\sigma) = W$" should be "with $P(\sigma) = V$ and $Q(\sigma) = W$".

- Page 302: the last two sentences of the third paragraph should be replaced with the following.

  Now since w_2 is a Littlewood-Richardson word and $x = y$, the word xyw_2 must have at least two more copies of x than it has copies of $x + 1$. But $z = x + 1$, so this is saying xyw_2 must have at least two more copies of $z - 1$ than it has copies of z. Therefore, the word zyw_2, which has one more z and one fewer $x = z - 1$ than xyw_2, must have at least as many copies of $z - 1$ as it has copies of z. But this contradicts the fact that zyw_2 has more copies of z than of $z - 1$.

- Page 304: there should not be a 2 at the end of Problem 10.9.

- Page 307: the sentence in Problem 10.12 should not be part of the last bullet point.

- Page 307: in the middle rhombus in Figure 10.20 the b and d should be switched.

- Page 315: in the third line from the bottom of the page, F should be F^n.

- Page 317: in the fourth line from the bottom of the page, $u_1, \ldots, u_n$ should be $\vec{u}_1, \ldots, \vec{u}_n$.

- Page 318: in Proposition A.18, "Suppose V is a vector space" should be "Suppose V is a vector space over $\mathbb{Q}$ or $\mathbb{R}$".

- Page 319: in the first line of the first displayed sequence of equalities, u_k should be $\vec{u}_k$.

- Page 319: in line 10, "and $w =$" should be "and $\vec{w} =$".

- Page 320: in the second line, "function" should be "bilinear map".

- Page 321: in Problem A.12, $\vec{w}, \ldots, \vec{w}_n$ should be $\vec{w}_1, \ldots, \vec{w}_n$.

- Page 333: in line 9, "of the form $1, j$" should be "of the form $(1, j)$".

- Back cover: "the involution Ω" should be "the involution ω".